CORE MATHEMATICS (C) UNIT 1 TEST PAPER 10

1. Express $\dfrac{2\sqrt{3}}{5-3\sqrt{3}}$ in the form $a+b\sqrt{3}$, where a and b are rational numbers. [4]

2. Find an equation of the circle with centre (2, –3) which passes through the point (7, 9). [4]

3. Given that $y = (2x-5)^2 - (x-4)^2$, find the integers c and d such that $y = c((x-d)^2 - 1)$. [5]

4. $f(x) \equiv 3x + \dfrac{1}{6x} - 4$.

   (i) Find the values of x for which $f'(x) = 0$, giving the answers in surd form with rational denominators. [4]

   (ii) Find the second derivative of $f(x)$ with respect to x. [2]

5. (i) By completing the square, find the roots of the equation $x^2 - 4kx + (5+k) = 0$, giving the values of x in terms of k. [4]

   (ii) Find the set of values of k for which the roots are real and distinct. [4]

6. Solve the simultaneous equations

$$2x - 3y = 1,$$

$$4x^2 - 9y^2 = 1 - 4x + 9y.$$ [8]

7. The straight line $y = mx + n$ is parallel to $4x - 2y = 5$ and passes through the point (1, 7).

The straight line $y = px + q$ is perpendicular to $6x + 3y = 4$ and passes through the point (−1, 5).

(i) Find the values of m, n, p and q. [6]

(ii) Find the coordinates of the point where the lines $y = mx + n$ and $y = px + q$ intersect. [3]

8. $f(x) \equiv x^3 - 11x^2 + 10x.$

(i) Factorise $f(x)$ completely. [3]

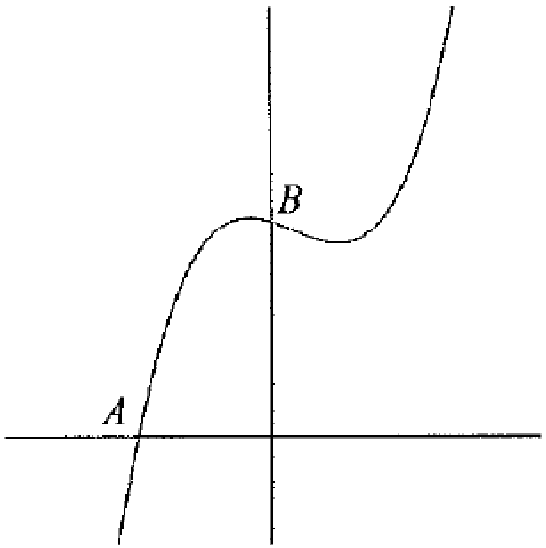
(ii) Find the gradient of the curve $y = f(x)$ at the origin. [3]

(iii) Find the coordinates of the points where the graph of $y = f(x + 3)$ crosses the x-axis. [4]

9. The diagram shows the curve C with equation

$$y = x^3 - x^2 - x + 10.$$

C cuts the x-axis at A $(a, 0)$, where a is an integer between -5 and 0, and the y-axis at B $(0, b)$.



(i) Find the values of a and b. [3]

(ii) Find the coordinates of the turning points of C, identifying each as a maximum or a minimum. [6]

The tangent to the curve at A and the normal to the curve at B meet at P.

(iii) Find the coordinates of P. [9]

CORE MATHS 1 (C) TEST PAPER 10 : ANSWERS AND MARK SCHEME

1.	$\dfrac{2\sqrt{3}}{5-3\sqrt{3}} = \dfrac{2\sqrt{3}(5+3\sqrt{3})}{(5-3\sqrt{3})(5+3\sqrt{3})} = \dfrac{18+10\sqrt{3}}{25-27} = -9-5\sqrt{3}$	M1 A1 A1 A1	4
2.	Radius = 13 $\quad (x-2)^2 + (y+3)^2 = 169$	M1 A1 M1 A1	4
3.	$y = (2x-5+x-4)(2x-5-x+4) = (3x-9)(x-1) = 3(x^2-4x+3)$	M1 A1	
	$= 3((x-2)^2-1) \quad c = 3,\ d = 2$	M1 A1 A1	5
4.	(i) $f'(x) = 3 - \dfrac{1}{6x^2} = 0$ when $x^2 = \dfrac{1}{18} \quad x = \pm\dfrac{\sqrt{2}}{6}$	M1 A1 M1 A1	
	(ii) $f''(x) = \dfrac{1}{3x^3}$	M1 A1	6
5.	(i) $(x-2k)^2 - (4k^2-k-5) = 0 \quad x = 2k \pm \sqrt{4k^2-k-5}$	M1 A1 M1 A1	
	(ii) $4k^2-k-5 > 0 \quad (4k-5)(k+1) > 0 \quad k < -1,\ k > 5/4$	M1 A1 M1 A1	8
6.	$(2x-3y)(2x+3y) = 1-4x+9y$, so $2x+3y = 1-4x+9y$	B1 M1 A1	
	$6x-6y = 1 \quad$ Also $6x-9y = 3$, so $y = -2/3 \quad x = -1/2$	M1 A1 M1 A1 A1	8
7.	(i) m = gradient of $4x-2y = 5$ so $m = 2 \quad$ Then $7 = 2+n$ so $n = 5$	M1 A1 A1	
	p = grad. perp. to $6x+3y = 4$ so $p = \frac{1}{2} \quad$ Then $5 = -\frac{1}{2}+q$ so $q = 11/2$	M1 A1 A1	

p = grad. perp. to $6x + 3y = 4$ so $p = \frac{1}{2}$ Then $5 = -\frac{1}{2} + q$ so $q = 11/2$ M1 A1 A1

(ii) $2x + 5 = \frac{1}{2}(x + 11)$ when $x = 1/3$ Intersect at (1/3, 17/3) M1 A1 A1 9

8. (i) $f(x) = x(x - 1)(x - 10)$ M1 A1 A1

(ii) $f'(x) = 3x^2 - 22x + 10 = 10$ when $x = 0$ M1 A1 A1

(iii) $f(x + 3) = 0$ when $(x + 3)(x + 2)(x - 7) = 0$ M1 A1

Points are (–3, 0), (–2, 0), (7, 0) M1 A1 10

9. (i) By trial, $y = 0$ when $x = -2$ $a = -2$, $b = 10$ M1 A1 B1

(ii) $dy/dx = 3x^2 - 2x - 1 = (3x + 1)(x - 1) = 0$ when $x = -1/3$, $x = 1$ M1 A1 A1

Turning points are $(-\frac{1}{3}, 10\frac{5}{27})$ max, (1, 9) min M1 A1 A1

(iii) At (–2, 0), gradient = 15 Tangent is $y = 15x + 30$ M1 A1 A1

At (0, 10), gradient = –1 Normal is $y = x + 10$ M1 A1 A1

At P, $14x = -20$ $x = -10/7$ P is (–10/7, 60/7) M1 A1 A1 18